Tentamen EMC 9-6-1999

a) $\frac{1}{Z'} = j\omega C + \frac{1}{R} = \frac{1+j\omega RC}{R} \rightarrow Z' = \frac{R}{1+j\omega RC}$

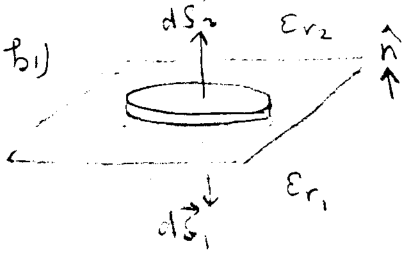
$Z = \frac{1}{j\omega C} + Z' = \frac{1}{j\omega C} + \frac{R}{1+j\omega CR} = \frac{1+2j\omega RC}{j\omega C(1+j\omega RC)} = \frac{1+2j\omega RC}{j\omega C - \omega^2 RC^2}$

b) $V_A = \frac{Z'}{Z} V_0 = \frac{R}{1+j\omega RC} \cdot \frac{j\omega C(1+j\omega RC)}{1+2j\omega RC} V_0 = \frac{j\omega RC}{1+2j\omega RC} V_0$

c) $V_A = |V_A| \cos(\omega t + \varphi)$ met

$|V_A| = \frac{\omega RC V_0}{\sqrt{1+4\omega^2 R^2 C^2}}$ en $\varphi = \frac{\pi}{2} - \arctan 2\omega RC$

2a) lineair: d.w.z. ε_r hangt niet af van het E-veld
isotroop: dwz ε_r is onafhankelijk van de richting van de kristal assen t.o.v. het E-veld.
homogeen dwz ε_r is overal in het materiaal hetzelfde

b1) 

Neem platte doos met 2 grote zijvlakken aan weerszijden van grensvlak. De doos is zo plat dat deze 2 zijvlakken (nagenoeg) volledige oppervlak v.d. doos vormen.

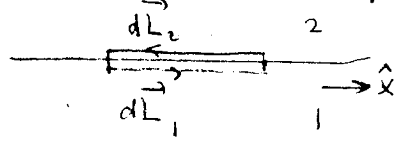
$\oint \vec{D} \cdot d\vec{S} = Q_c \longrightarrow \vec{D}_1 \cdot d\vec{S}_1 + \vec{D}_2 \cdot d\vec{S}_2 = Q_c \rightarrow$

$d\vec{S}_1 = -\hat{n}\, dS$ $\quad D_{1n} = \vec{D}_1 \cdot \hat{n}$ $\quad -D_{1n}\, dS + D_{2n}\, dS = \sigma_c\, dS$

$d\vec{S}_2 = \hat{n}\, dS$ $\quad D_{2n} = \vec{D}_2 \cdot \hat{n}$ $\quad \rightarrow D_{2n} - D_{1n} = \sigma_c \longrightarrow$

Als $\sigma_c = 0 \rightarrow \varepsilon_{r1} E_{1n} = \varepsilon_{r2} E_{2n}$ $\quad \boxed{\varepsilon_0 \varepsilon_{r2} E_{2n} - \varepsilon_0 \varepsilon_{r1} E_{1n} = \sigma_c}$

b2) Neem rechthoekige contour langs het grensvlak z.d.d. de helft is in medium 1 en de helft in medium 2.



$\oint \vec{E} \cdot d\vec{L} = 0$

$\vec{E}_1 \cdot d\vec{L}_1 + \vec{E}_2 \cdot d\vec{L}_2 = 0$ $\quad d\vec{L}_1 = dL\, \hat{x}$ $\quad E_{1t} = \vec{E}_1 \cdot \hat{x}$

$E_{1t}\, dL - E_{2t}\, dL = 0$ $\quad d\vec{L}_2 = -dL\, \hat{x}$ $\quad E_{2t} = \vec{E}_2 \cdot \hat{x}$

$\rightarrow \boxed{E_{1t} = E_{2t}}$

Tentamen EM2 9-6.1999

3a) $\oint_c \vec{H}\cdot d\vec{L} = NI$ (langs gestippelde contour

$H_1 \pi R + H_2 \pi R = NI \rightarrow \frac{B}{\mu_0\mu_{r1}}\pi R + \frac{B}{\mu_0\mu_{r2}}\pi R = NI$

$B\frac{\pi R}{\mu_0}\left(\frac{\mu_{r1}+\mu_{r2}}{\mu_{r1}\mu_{r2}}\right) = NI \rightarrow B = \frac{\mu_0\mu_{r1}\mu_{r2}NI}{\pi R(\mu_{r1}+\mu_{r2})}$

b) $M_1 = \frac{B}{\mu_0} - H_1 = \frac{B}{\mu_0} - \frac{B}{\mu_0\mu_{r1}} = \frac{B(\mu_{r1}-1)}{\mu_0\mu_{r1}} = \frac{\mu_{r2}(\mu_{r1}-1)NI}{\pi R(\mu_{r1}+\mu_{r2})}$

Zo ook $M_2 = \frac{\mu_{r1}(\mu_{r2}-1)NI}{\pi R(\mu_{r1}+\mu_{r2})}$

4a) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ $\quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}\cdot\vec{B} = 0$ $\quad \vec{\nabla}\times\vec{H} = \vec{J}_c + \frac{\partial\vec{D}}{\partial t}$

b) Vacuum $\rightarrow \rho = 0$, $J_c = 0$ $\rightarrow$ $\vec{\nabla}\cdot\vec{E} = 0$ $\quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{D} = \varepsilon_0\vec{E}$ $\quad \vec{B} = \mu_0\vec{H}$ $\quad \vec{\nabla}\cdot\vec{B} = 0$ $\quad \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$\vec{\nabla}\times\vec{\nabla}\times\vec{A} = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A}$

$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) \rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

$\rightarrow -\vec{\nabla}^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \rightarrow \vec{\nabla}^2\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$

c) Vlakke golf $\quad E_y = E_{y0}\cos(kx-\omega t)$ $\quad \vec{E} = E_y\hat{y}$

$B_z = B_{z0}\cos(kx-\omega t)$ $\quad \vec{B} = B_z\hat{z}$

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \rightarrow (\vec{\nabla}\times\vec{E})_z = -\frac{\partial B_z}{\partial t} \rightarrow \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$

$\rightarrow -kE_{y0}\sin(kx-\omega t) = -\omega B_{z0}\sin(kx-\omega t) \rightarrow kE_{y0} = \omega B_{z0} \rightarrow \frac{E_{y0}}{B_{z0}} = \frac{\omega}{k} = c$

$\rightarrow E_{y0} = cB_{z0} \rightarrow Z = \frac{E}{H} = \mu_0\frac{E_{y0}}{B_{z0}} = \mu_0 c =$

$= 4\pi\times10^{-7}\ 3\times10^{+8} = 30\times4\pi = 377\,\Omega$